Remote Sensing Satellite Constellation Design Based on Repeat Ground Track Orbits Properties

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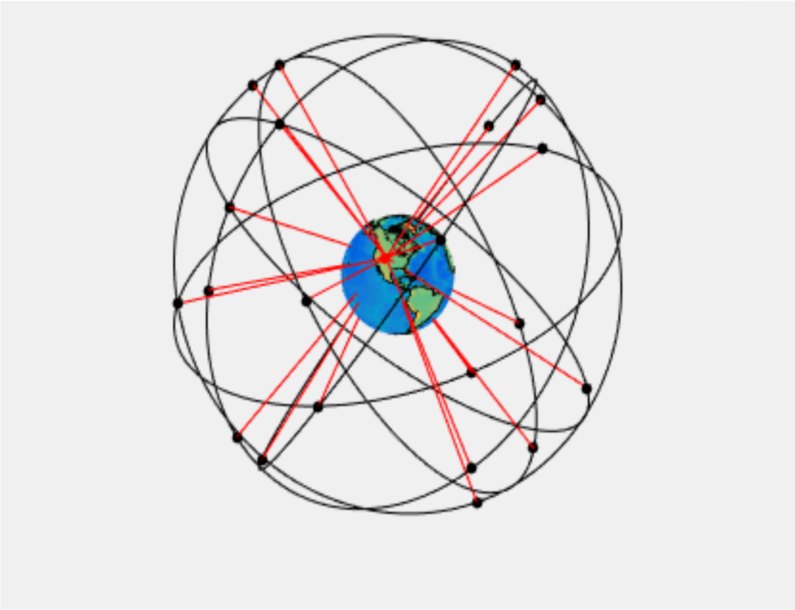
Presentation Outline

- Distributed Satellite Systems
- Earth Observation Satellite Constellation
- Numerical Earth Coverage Analysis
- Satellite Ground Track Properties
- Remote Sensing Mission Design Approach
- Constellation Design Example

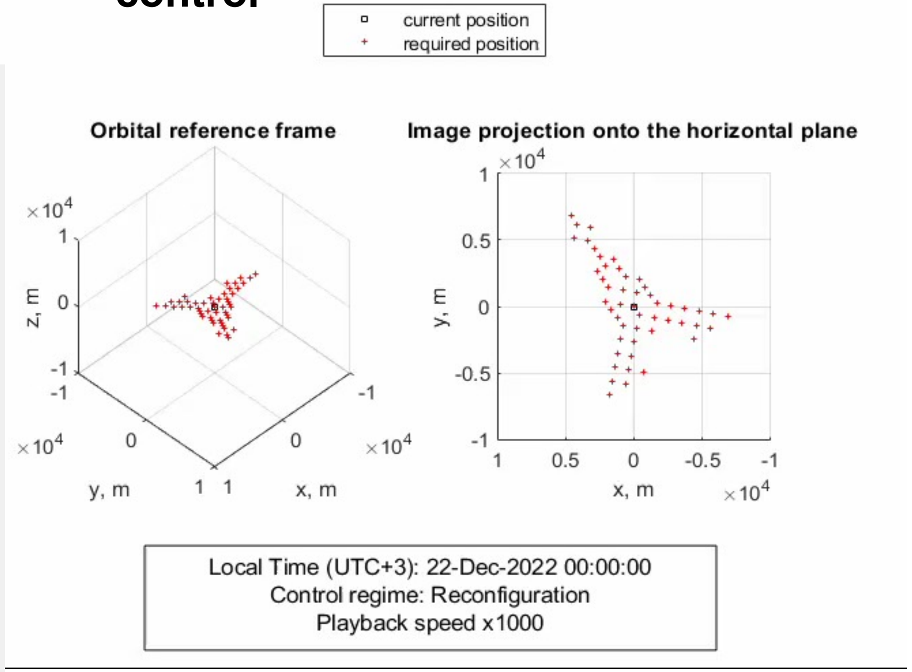
Distributed Space Systems

Constellation: big ISD, no relative motion control; coordinated from a ground control center



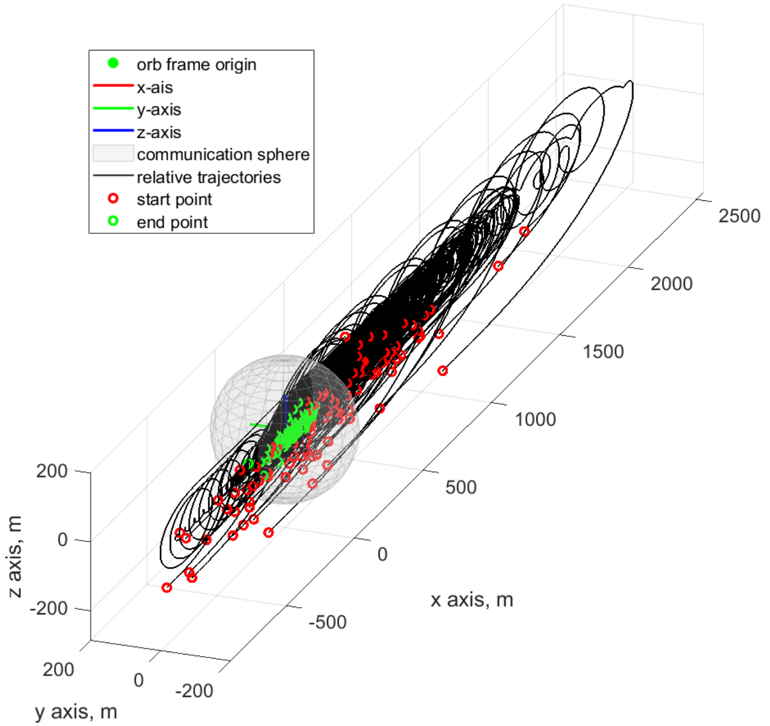
GPS constellation

Formation: small ISD, relative motion control, fixed orbital configuration, centralized control



Solar Sail-Equipped Satellite Formation Flying for graphic image demonstration

Swarm: small ISD, relative motion control but without fixed relative positions, decentralized control



Femtosatellites swarm

* ISD – inter-satellite distance

Earth Observation Satellite Constellation

- Earth Observation System is characterized by imaging resolution or ground sample distance (GSD) and Earth coverage Figures of Merits (FOMs)
- Most of the EO systems are discontinuous coverage constellations comprising LEO satellites at repeating orbits
- Satellite might be equipped with optical or radar telescopes of a given angular resolution α, field of view angle γ. Some of the satellites can be steerable, thus increasing satellite access area or field of regard (FOR)
- Optical telescopes has limitation in imaging in terms of lighting conditions defined by corresponding critical sun elevation at observer location

Problem Statement. Given telescope angular resolution θ_r, find RGT SSO circular orbit radius R and corresponding minimum number of satellites n_{sats} to provide coverage of an area of interest (AOI) with a certain revisit time T_{rev} and a required image quality as per National Image Interpretability Rating Scales NIIRS.

Table 1. National Image Interpretability Rating Scales (NIIRS)

NIIRS	Resolution	EO
0		Un-interpretable Image
1	> 9.0 m	Distinguish between roads, runways
2	4.5 to 9.0 m	Detect parking garage
3	2.5 to 4.5 m	Detect parking space
4	1.2 to 2.5 m	Recognize vehicle types
5	0.8 to 1.2 m	Recognize vehicle configurations
6	0.4 to 0.8 m	ID vehicle classes
7	0.2 to 0.4 m	ID vehicle tires
8	0.1 to 0.2 m	ID driver
9	< 0.1 m	ID vehicle plates

Coverage Figures of Merits (FOMs)

- **Revisit Time or Coverage Gap Statistics**

The length of breaks in coverage for a given point on the simulation grid.
Crucial to consider maximum and average revisit time

- **Response Time Statistics**

The response time is the time from when we receive a random request to observe a point until we can observe it but any satellite within constellation. Typically, average response time is considered.

- **Specific Access**

Number of an Earth grid point accesses with a constellation satellite per unit time (day, month)

- **Percent Coverage**

The number of times that point was covered by one or more satellites divided by the total number of simulation time steps

Reference: James Richard Wertz. Orbit & Constellation Design & Management: Spacecraft Orbit and Attitude Systems. Microcosm Press, 2001

Satellite Payload

- Satellite swath dimensions are defined by the telescope field of view and orbit
- The footprint geometry is defined as:
  - Optical telescope (circular conic): field of view angle θ_f
  - Radar systems: elevation angle range and exclusion angles
- The ground sample distance (GSD) defining the resolution of the images strongly depends on the telescope look angle $\theta_l$measured from the nadir direction
- For optical telescopes, the minimum GSD is achieved when pointing at nadir direction, i.e., $\theta_l = 0$. For a given look angle θ_l, GSD can be found as follows

$$GSD = (\beta_{out} + \beta_{in}) \cdot R_{\oplus},$$

$$\beta_{in} = \beta_c - \frac{\pi}{2} - (\theta_l - \frac{\theta_r}{2}) - \arccos(\frac{R_s}{R_{\oplus}} \sin(\theta_l - \frac{\theta_r}{2})),$$

$$\beta_{out} = \frac{\pi}{2} - (\theta_l + \frac{\theta_r}{2}) - \arccos(\frac{R_s}{R_{\oplus}} \sin(\theta_l + \frac{\theta_r}{2})) - \beta_c,$$

$$\beta_c = \frac{\pi}{2} - \theta_l - \arccos(\frac{R_s}{R_{\oplus}} sin(\theta_l)).$$

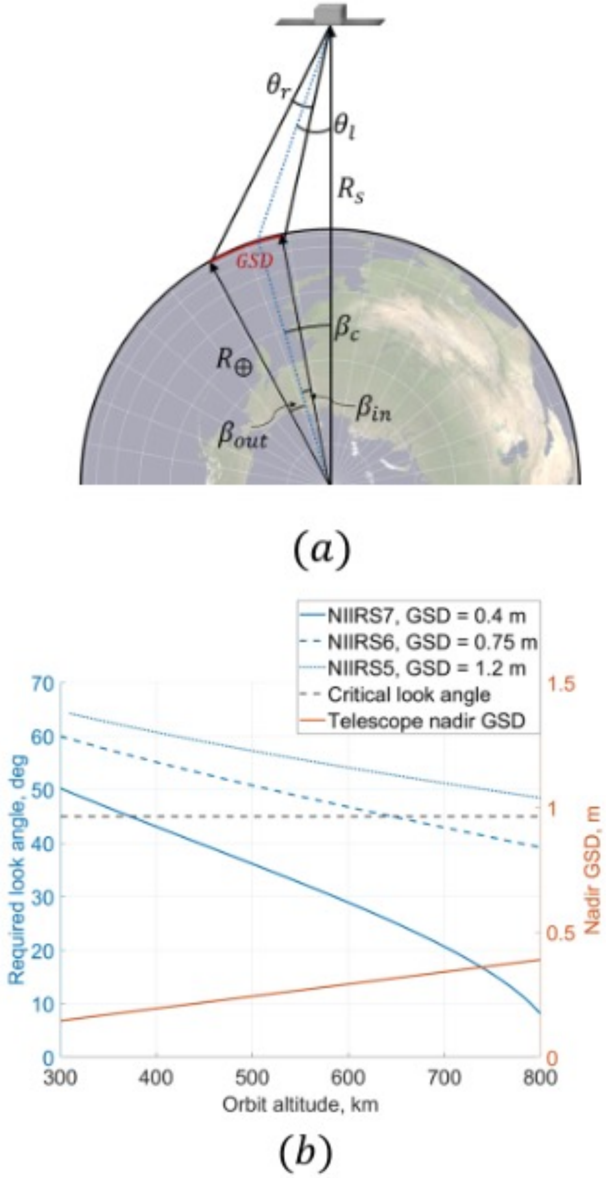


FIGURE 1. a) Sensor geometry, b) Nadir GSD(h) & Required look angle $\theta(h)$ to achieve different GSD or NIIRS classes.

Orbital Motion Dynamics

- J2-perturbed satellite orbital motion dynamics is considered in the study for mission design purposes. Satellites are supposed to operate in near circular orbits

- For the constellation design purpose the only secular change of orbital elements is taken into account for orbit propagation

$$\begin{cases} \dot{a} = 0, \\ \dot{e} = 0, \\ \dot{i} = 0, \\ \dot{\Omega} = -\frac{C}{a^{7/2}} \cos i, \\ \dot{\omega} = -\frac{C}{a^{7/2}} (\frac{5}{2} sin^2 i - 2), \\ \dot{M} = n - \frac{C}{a^{7/2}} (\frac{3}{2} sin^2 i - 1), \end{cases}$$

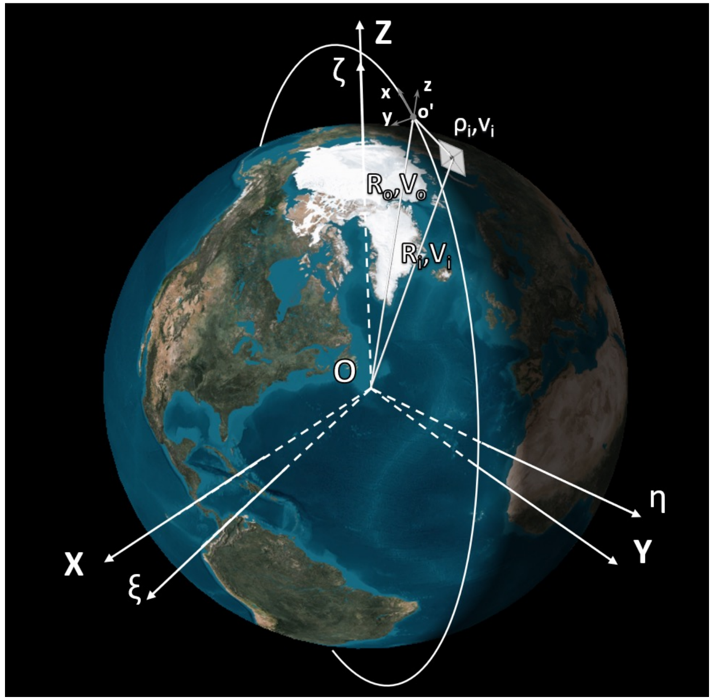
$$\begin{cases} \mathbf{r}(t) = A^z(\Omega(t)) \cdot A^x(i(t)) \cdot A^z(u(t)) \cdot \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}, \\ \mathbf{v}(t) = A^z(\Omega(t)) \cdot A^x(i(t)) \cdot A^z(u(t)) \cdot \begin{bmatrix} 0 \\ \sqrt{\frac{\mu_\oplus}{a}} \\ 0 \end{bmatrix} \end{cases}$$

where $C = \frac{3}{2} \mu_\oplus^{1/2} R_\oplus^{eq} J_2$, and n is the Keplerian orbit mean motion.

A^z and A^x represents rotation matrix around z and x axis, $u = \omega + M$ is argument of latitude. The order of matrix multiplication is defined by the rule of intrinsic rotations - each rotation matrix is defined in new coordinates.

- For the J2-perturbed dynamics model, Earth and satellite nodal periods T_G^n, T_{sat}^n can be calculated as follows:

$$T_G^n = \frac{2\pi}{\omega_\oplus - \dot{\Omega}} \qquad T_{sat}^n = T_k \left[1 - \frac{3 J_2}{2} \left(\frac{\mathcal{R}_\oplus}{R_0} \right)^2 (3 - 4 \sin^2(i)) \right]$$



Referefe frames.
$OXYZ$ – Earth-centered inertial reference (ECI) frame,
$O\xi\eta\zeta$ – Earth-centered Earth-fixed (ECEF) reference frame,
$o'xyz$ – orbital reference frame

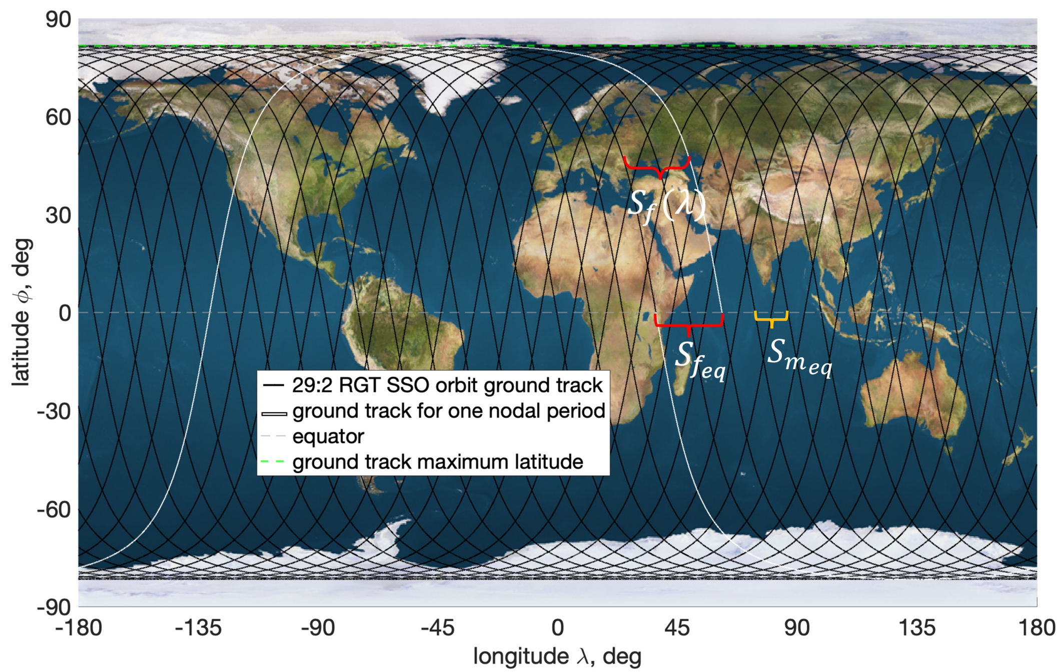
Satellite Ground Track

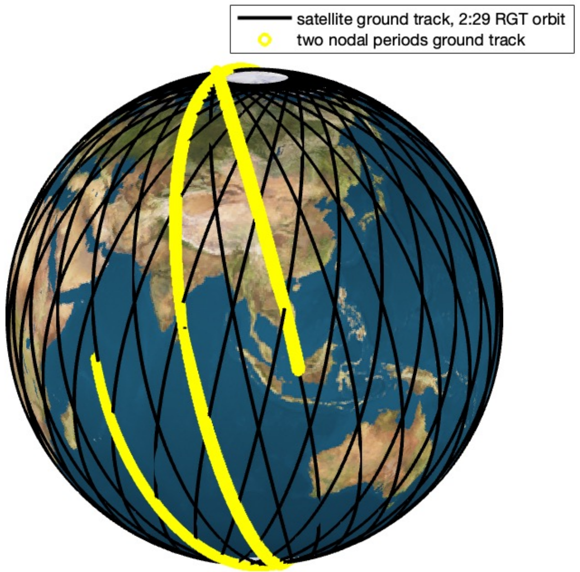
- Satellite ground track is a locus of sub-satellite points or points on Earth having the same unit vectors $\boldsymbol{e}_{sat}^{E}$ as the satellite position vector $\boldsymbol{R}_{sat}^{E}$ given in ECEF frame
- It is common to consider 2D ground track plots, where each point is defined by corresponding latitude ϕ and longitude λ
- Fundamental interval S_F is a crucial parameter representing the longitudal shift of satellite after one nodal period

$$S_F = \left(\omega_{\oplus} - \dot{\Omega}\right) \cdot T_{sat}^{n}$$

- Minimum separation interval S_m is the is the minimum interval on equator of a constellation between two ground tracks
- Maximum ground track latitude ϕ_{max} is defined by satellite inclination as follows

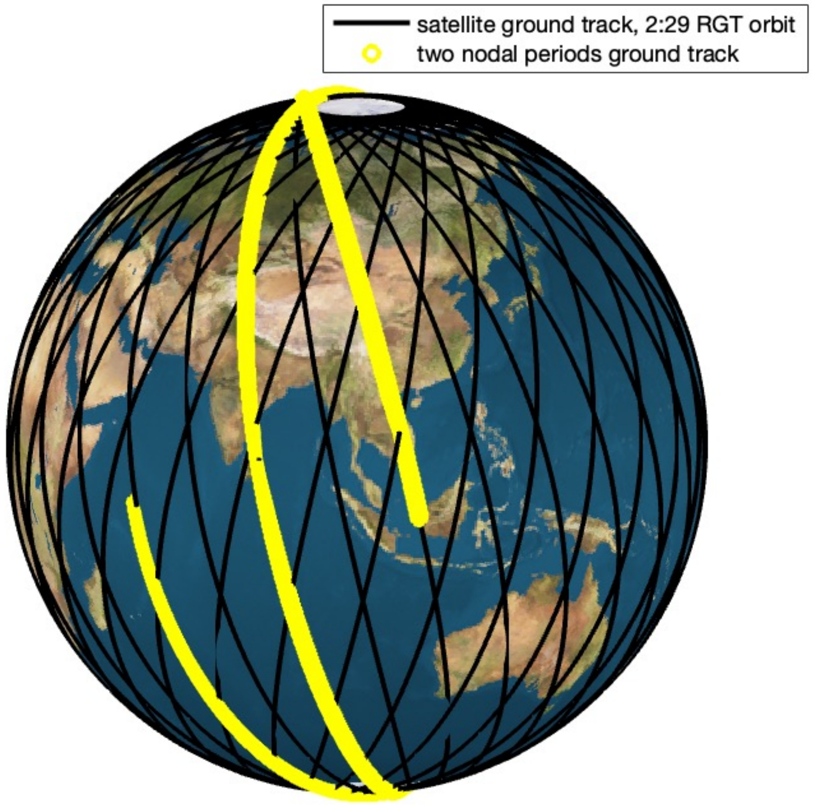
$$\sin(\phi_{max}) = \sin(i)$$

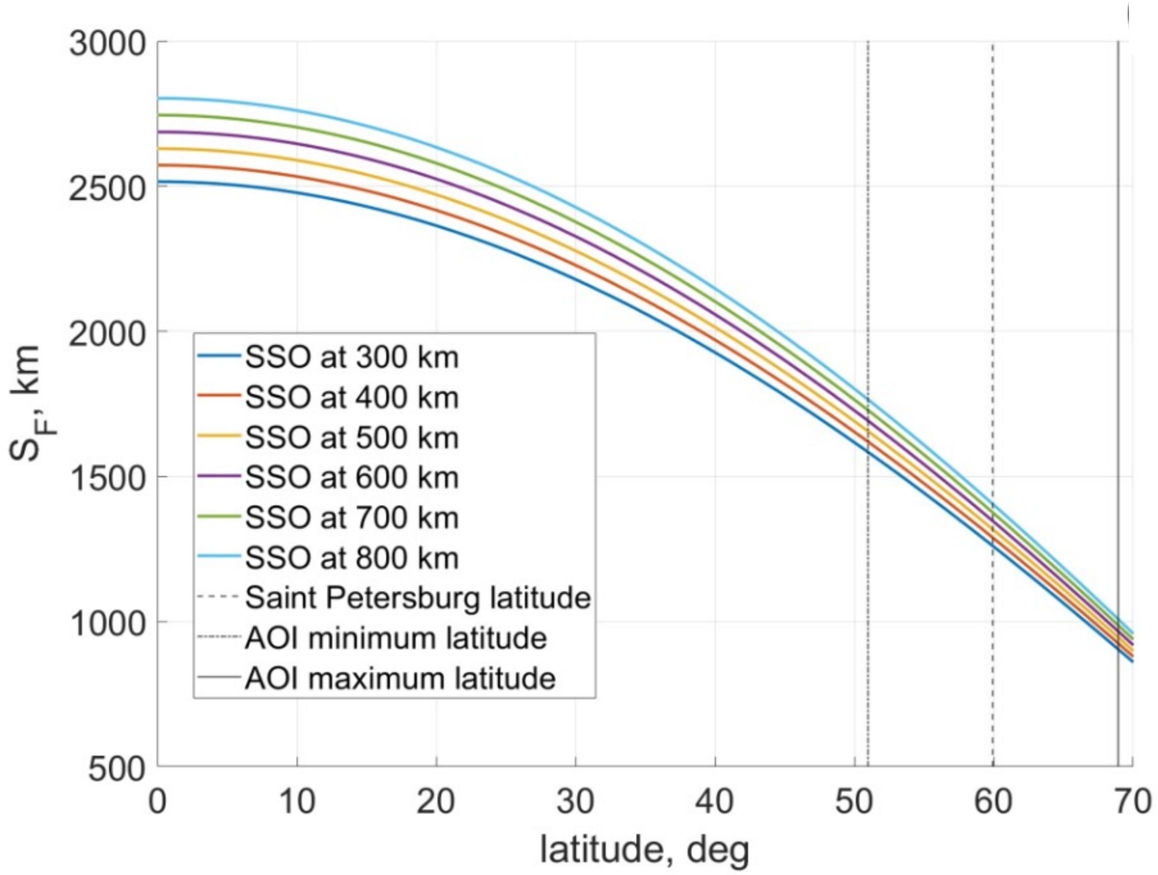




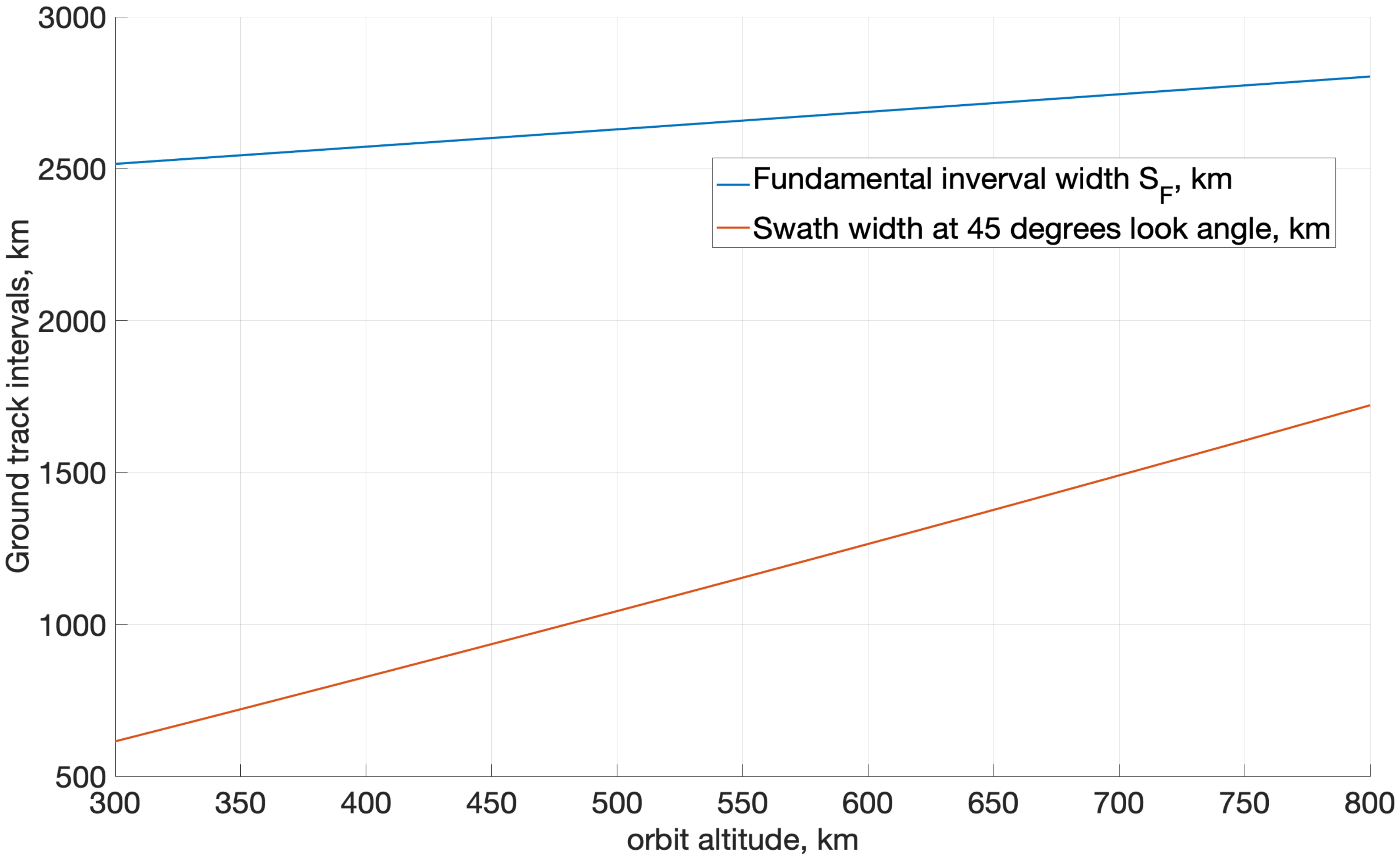
Fundamental Interval vs Latitude

- The fundamental interval S_F is decreasing with latitude
- To analyze how many S_F is needed to cover the full AOI, S_F at the AOI latitude must be considered



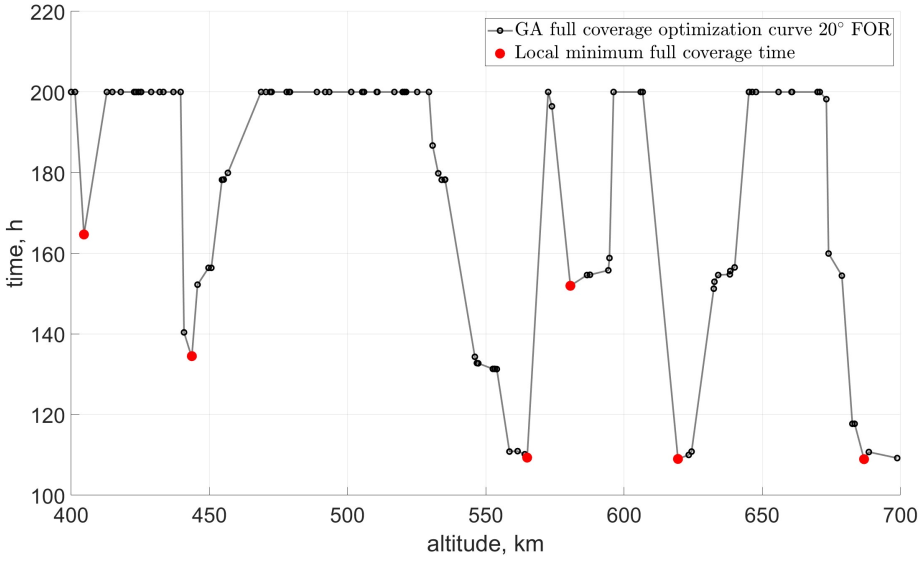


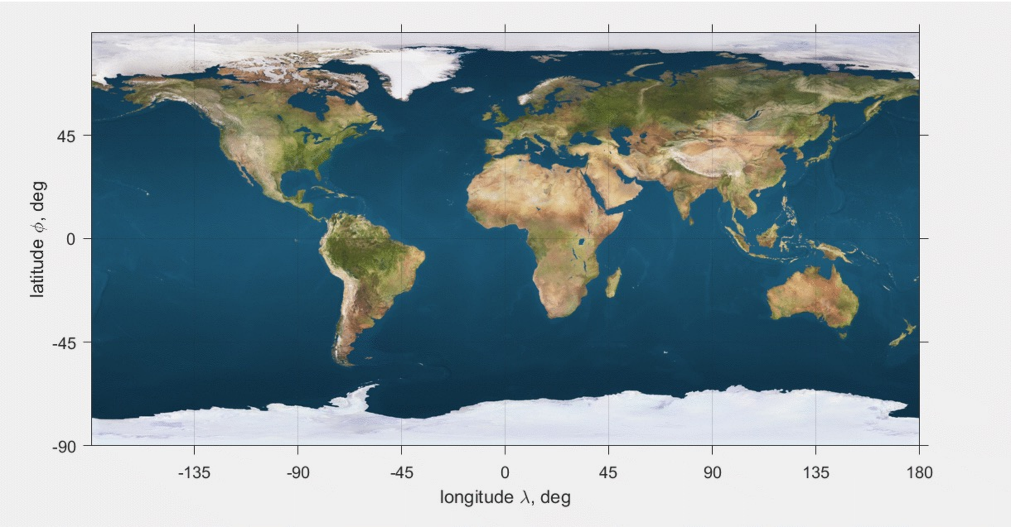
Fundamental Interval vs. Swath Width

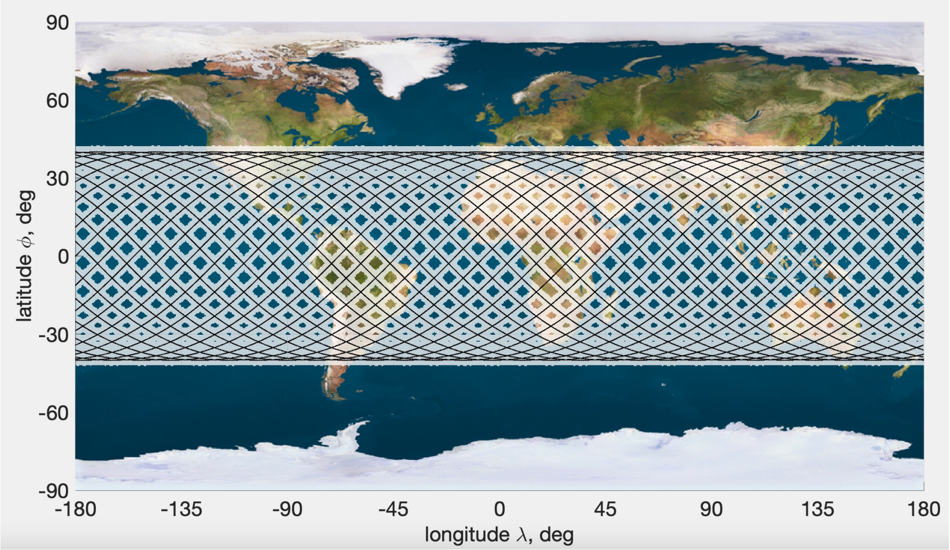


Numerical Analysis of the Earth Coverage

- Numerical Earth coverage analysis approach is the so-called point coverage, i.e. the coverage is being assessed by checking geometrical coverage conditions for all possible pairs of *i*-th satellite and *j*-th grid node
- Earth is considered as spherical. Earth grid node is made using Fibonacci spiral. Satellite state is being propagated by analytical J2 propagator
- A maximum revisit time was optimized for mid-inclination orbit by varying circular orbit altitude. Satellite field of view of 20 degrees was considered
- Surprisingly maximum revisit time curve has several picks. The picks correspond to the repeating orbits with insufficient swath width to ensure coverage of entire Earth







Repeat Ground Track Orbits

- An RGT orbit has commensurability of its nodal period T_{sat}^{n} with nodal period of the Earth self-rotation T_{sat}^{G} and is typically defined by D days to repeat within R revolutions

$$T_G^n = \frac{2\pi}{\omega_{\oplus} - \dot{\Omega}} \qquad T_{sat}^n = T_k \left[1 - \frac{3J_2}{2}\left(\frac{\mathcal{R}_{\oplus}}{R_0}\right)^2 (3 - 4\sin^2(i))\right]$$

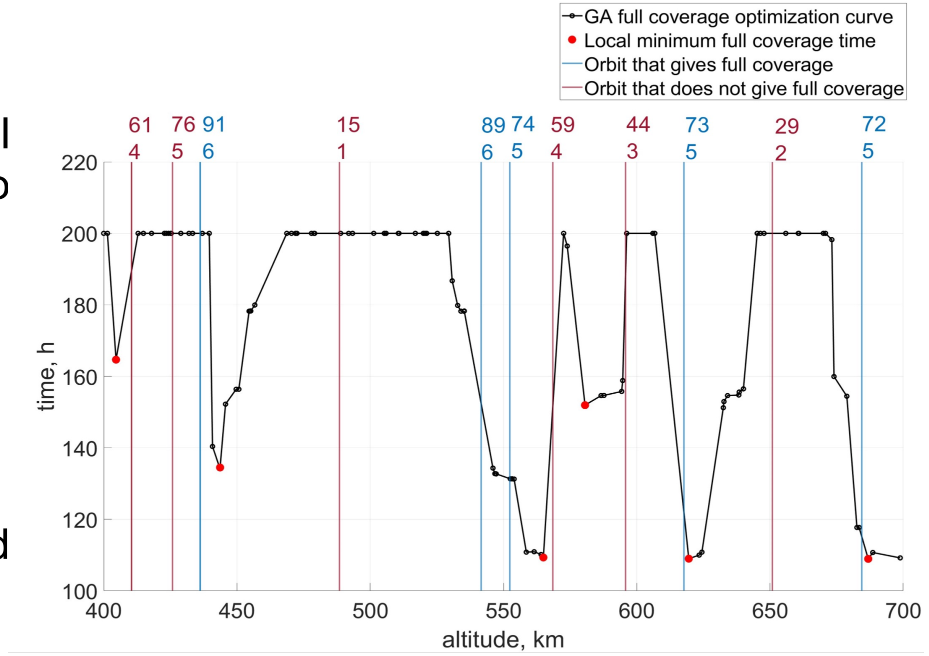
$$R \cdot T_{sat}^n = D \cdot T_G^n$$

- For a repeat ground track orbit, ground track intervals S_F, S_m are calculated as follows

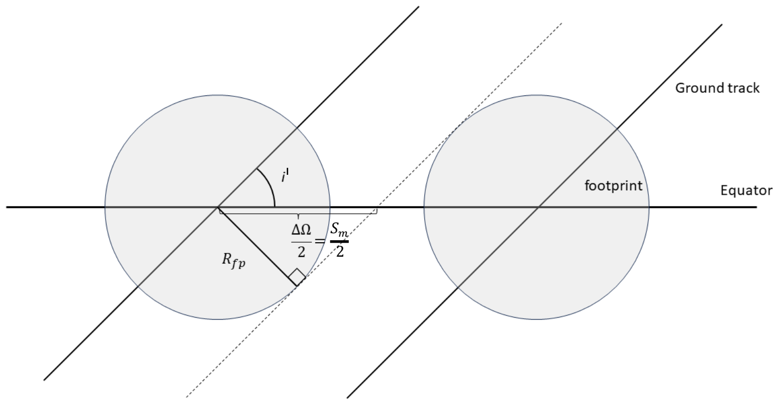
$$S_F = \frac{2\pi}{Q}, S_m = \frac{S_F}{D}, \mathrm{Q} = \frac{\mathrm{R}}{D} = I + \frac{K}{D}$$

- For a wide enough satellite instrument swath width S_w the maximum revisit time T_{rep} can be equal to the time of orbit repetition D. The swath width S_w should satisfy the following expression

$$S_w \geq S_m \cdot \sin(i'),\ i' = \frac{\sin(i)}{\cos(i) - \frac{1}{Q}}$$



GA based optimization of satellite orbit using propulsion size of 100 and 2 generation, FOR = 20 deg



Constellation Design Approach

- The main objective of this study is to identify the minimum required number of satellites n_{sats} to achieve consecutive coverage of all points in a specific region multiple times within a day

- Let's consider a single plane constellation design yielding a complete coverage of an AOI per day in this study. To get multiple accesses per day, the configuration of the single plane constellation could be repeated with multiple identical orbital planes shifted by RAAN with respect to each other.

- The minimum number of satellites to ensure full coverage of the entire area of interest can be found as follows:

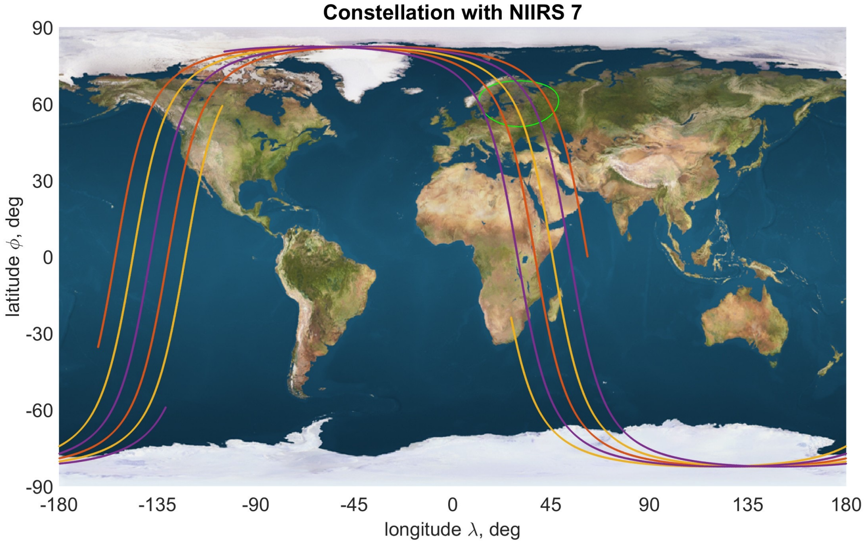
$$n_{sats} = \left\lceil \frac{S_F(\phi_{min})}{\tilde{S}_w} \right\rceil, \qquad \tilde{S}_w = S_w \cdot \frac{1}{\sin(i')},$$

- In order to provide evenly spaced satellites' ground tracks within fundamental interval S_F , the distance between adjacent ground tracks is found as follows

$$\Delta\lambda = \frac{S_F(0)}{n_{sats}},$$

- To yield ground track shift of $\Delta\lambda$ either of the following conditions shall be met:

$$\begin{cases} \Delta\Omega = \Delta\lambda, \\ \Delta MA = \Delta\lambda \cdot Q. \end{cases}$$



Example and Numerical Results

- To verify the constellation design approach, an Area of Interest (AOI) can be selected to determine the number of satellites required to achieve specific FOMs
- In this example, Saint Petersburg (latitude $\phi = 59.94°$, longitude $\lambda = 30.31°$) with its circumference of radius of 1000 km is considered as an AOI
- The goal of the constellation is to get consecutive coverage of the full AOI for a given revisit time
- The problem is solved as follows:
    - Find the number of satellites n_{sats} required to get a consecutive coverage of AOI, and specify the orbital configuration of the satellites in one plane
    - Determine the number of orbital planes n_{planes} required to achieve the requested revisit time
    - The configuration defined in the first step is followed in every orbital plane.
    - The total number of satellites N_{sats} is:

$$N_{sats} = n_{sats} \cdot n_{planes}$$

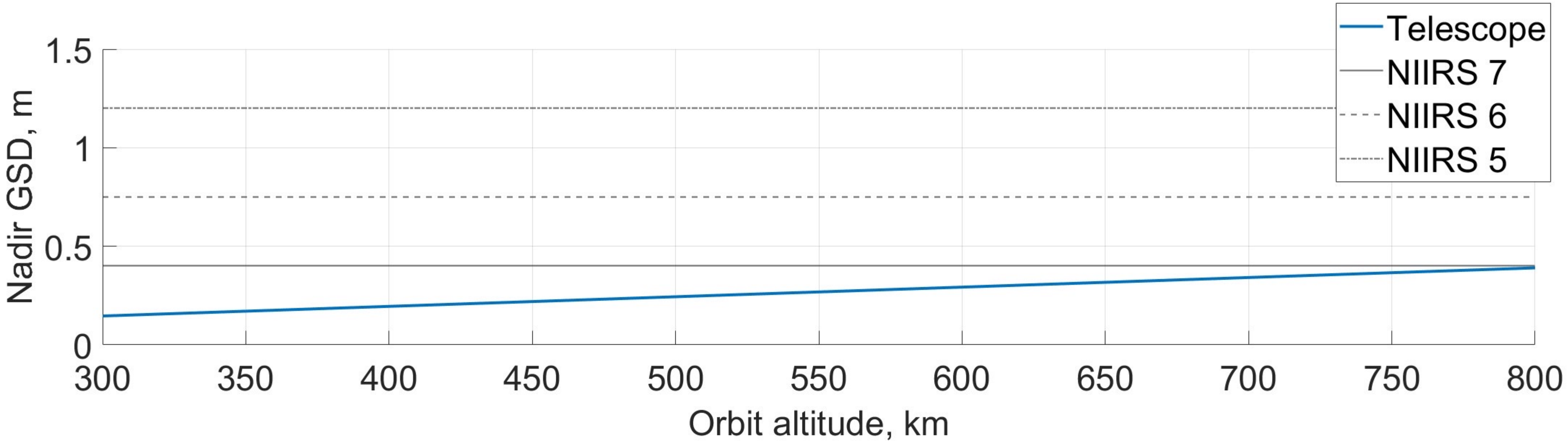
Area of interest (AOI):
Saint-Petersburg with 1000 km radius.

Example and Numerical Results

- High resolution optical camera is considered with diameter $\mathrm{D} = 1\ m$
- The angular resolution of the telescope is calculated using diffraction limit as follows (blue light wavelength is used, $\lambda = 400\ nm$):

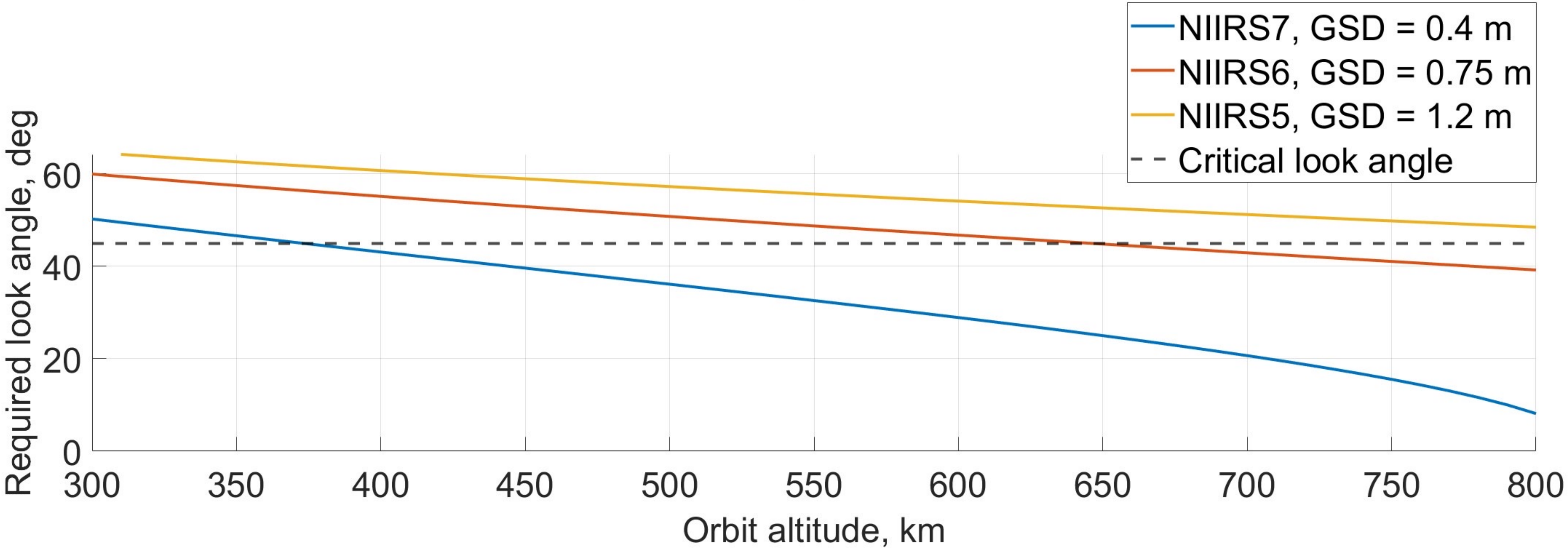
$$\theta_r = \frac{1.22\ \lambda}{D} = 4.88 \cdot\ 10^{-7}$$

- Nadir GSD versus altitude:



Example and Numerical Results

- Based on the image resolution requirements, different GSD can be achieved by varying the look angle
- Higher look angle provides an access to a bigger ground area but with lower image resolution

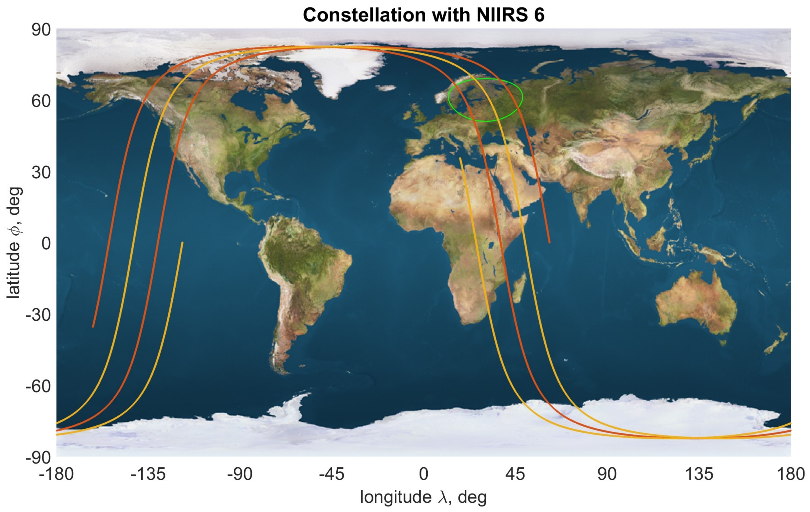


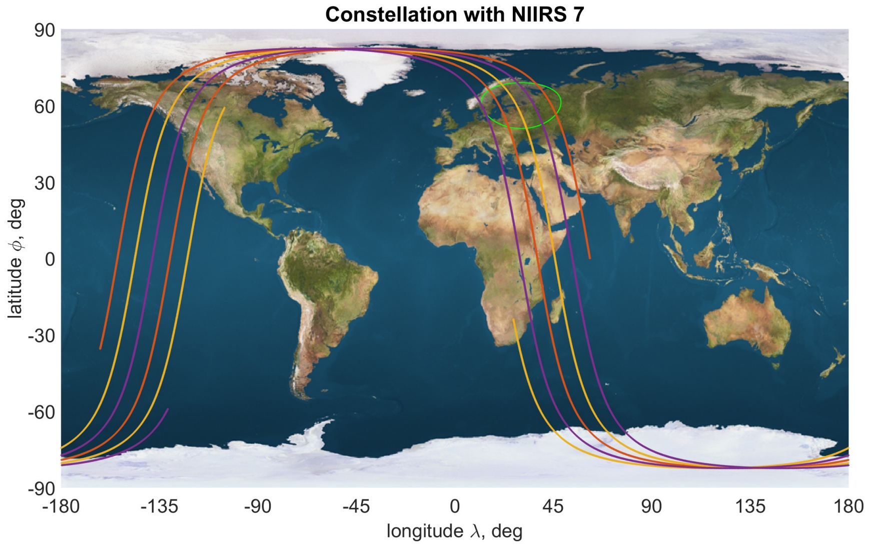
Example and Numerical Results

- Sun-synchronous repeat ground track orbit (2:29) at altitude 727.1 km is selected to calculate the number of satellites required to achieve the consecutive coverage of the AOI
- At this altitude, the nadir GSD is equal to 0.35 m, so with this telescope the best image will have NIIRS7 scale with maximum look angle 17.4 degrees
- To reduce the required number of satellites, NIIRS6 can be considered with a higher look angle

TABLE 1. Constellation parameters

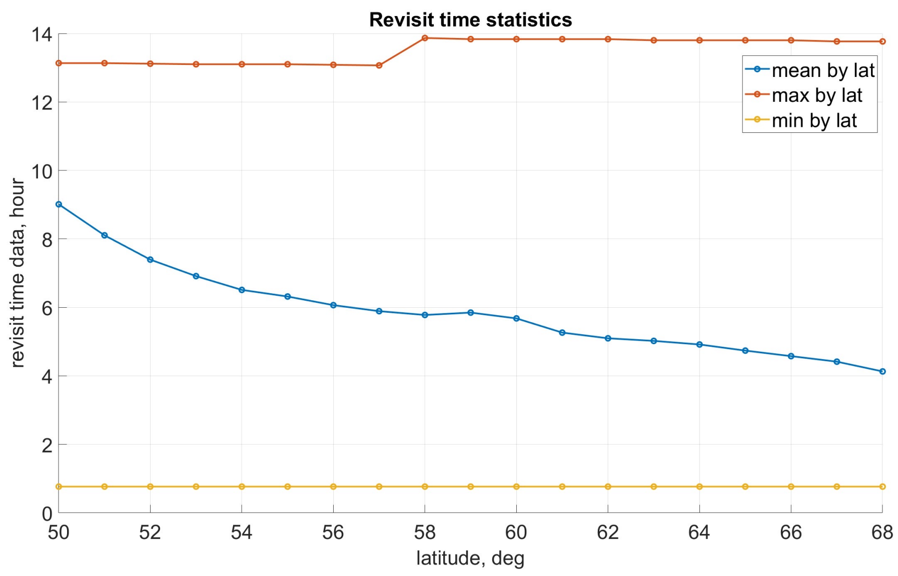
Parameters	NIIRS6	NIIRS7
Number of satellites	2	4
Swath width S_w, km	1357.8	458.4
Maximum look angle θ, degrees	41.6	17.4
Minimum swathes overlapping, %	35.98	5.19
Mean anomaly separation ΔMA, degrees	180	90

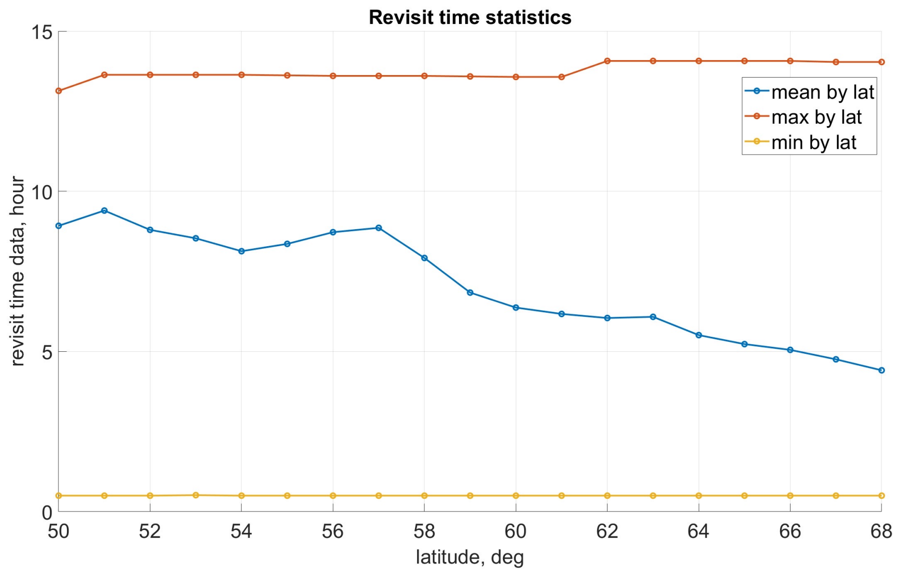




Example and Numerical Results

- The approach to design a constellation is verified using revisit time analysis
- For a single plane constellation, the satellites will have maximum two access per day to the AOI
- The results show that for one day every point within the AOI has revisit time $\approx 12\ hours$
- This means that with the designed planer constellation, the points within AOI are covered consecutively





Conclusion

- An approach was desgined to find a minimum number of satellites to ensure proper coverage FOM's of an Earth observation constellation with given telescope parameters
- The method can be applied for a single and multiple plane constellations by proper satellite placements with RAAN of MA shift yeilding proper ground track longitudal shift
- Further work necessitates consideration of
    - Earth oblateness in coverage geometry model,
    - Numerical analysis of constellation coverage in the complex force model,
    - Coverage properties of constellation consisting of satellites with different payload

Relevant references:

[1] James Richard Wertz. Orbit & Constellation Design & Management: Spacecraft Orbit and Attitude Systems. Microcosm Press, 2001.

[2] Ortore Emiliano, Cinelli Marco, Circi Christian, A ground track-based approach to design satellite constellations, Aerospace Science and Technology, 2017

[3] Luo X. et al. Constellation design for earth observation based on the characteristics of the satellite ground track //Advances in Space Research. – 2017.

[4] Circi C., Ortore E., Bunkheila F. Satellite constellations in sliding ground track orbits //Aerospace Science and Technology. – 2014.